The Use of Sun, Quiet Sky and Earth Noise Measurements for Determining System Parameters, with Error Analysis

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Introduction and Background - The use of Sun noise for the evaluation of microwave stations having elevation control on the antenna is very common. The technique consists of making two noise measurements, with the antenna pointed at the Sun and at the "quiet" sky. This has been discussed at length [1-6][1] in articles going back to at least 1960 [7]. The biggest change that has occurred in the last few years is the added availability of accurate and current Sun noise data over the Internet. This permits accurate evaluation of the receiving parameter, *g/t*. However, these ideas change slowly and "Sun Noise" measurements are still widely used without correction for the current solar flux.

Going one step further, we can make use of an additional measurement, the noise of a known temperature that fully encompasses the antenna beam. The usual such noise source is the Earth. This allows the determination of receiver noise temperature, t_R. and system noise temperature, *t* [5]. If the measurements are reasonably accurate, it is easy to then calculate the antenna gain, *g*, from the estimates of *g/t* and t.

In the following pages, this procedure is outlined, along with sources of data and methods for maximizing the accuracy of the measurements. Graphs are presented to simplify the calculations. Following this is an error analysis, allowing us to determine the sensitivities of the various estimated parameters to the measurements. Our estimates will never be a totally accurate, but we can have knowledge of the general magnitude of the errors. With care, the procedure of the three measurements will be shown to produce estimates of *g/t*, *t* and *g* to useful accuracy.

Each of the measurements and needed assumptions is discussed in the following sections. Those wanting to get a fast start with the measurements can go directly to the "Example" section, that will allow answers to be generated and even some "feel" for the method. However, it is highly recommended that you return to the detailed sections and do enough study to be sure all the assumptions are appropriate for your station.

Math Symbols - Some algebra is part of the subject of noise measurements. One element that can cause confusion is the use of dB to represent ratios. It is extremely important to know when the dB is being used, and when not! To minimize confusion, a ratio will be represented by a lower-case main letter and the same quantity in dB will use a capital main letter. There may be subscripts, but they will not switch between lower case and capitals. For example, y_{SQ} is the ratio of the noise power measured for the Sun and the quiet sky and Y_{SQ} is the same quantity expressed in dB.

The symbol K is the abbreviation for the absolute temperature unit Kelvins. It should not be confused with the lower-case version, k, which is Boltzmann's constant, defined later.

Solar Flux Determination - Fundamental to using the Sun as a signal generator is determining its output setting! Fortunately, this has become simpler with the availability of up-to-date measurements over the Internet.

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[1] Notes and references are at the end of the text. These are indicated by numbers inside square brackets.

To maximize the accuracy of the solar flux data, try to make the Sun-noise measurements near the time-of-day when the calibrated observatory data is available. Presently, the solar flux data is available from the Space Environmental Center (SEC) at http://www.sec.noaa.gov/ftpmenu/lists/radio.html. For instance, for those in North America, there is a daily set of data taken at Sagamore Hill, Massachusetts, at 1600 UTC. In addition, a 2800 MHz measurement is made at Penticton, British Columbia, at 1700 UTC. If your Sun measurements are made around this time of day, the Sun can be high in the sky as well as having current flux data available. The Penticton data can be correlated with the Sagamore Hill data to add to the confidence that the solar flux levels were steady during the period. As an example, on June 27, 2003 the following data was posted for Sagamore Hill and Penticton (there were also more sets of data from various observatories around the world).

```
 245 MHz  16 SFU
 410       35
 610       43
 1415      79
 2695     117
 2800     123  (Penticton data)
 4995     170
 8800     261
 15400    549
```

Radio bursts can occur that will create short-term changes in the solar flux data. This is particularly true below a few thousand MHz and can cause wild variations at frequencies such as 144 MHz. Again, Internet data is available to minimize errors due to these bursts. The SEC issues burst reports every half hour. As of this writing, these are available at http://www.sec.noaa.gov/ftpdir/lists/radio/radio_bursts.txt. The safest procedure is to check this data an hour or so after your Sun-noise measurements and discard data if a burst event has occurred. This also emphasizes the utility of making multiple measurements, spread over an hour or two, as many burst events are of relatively short duration.

The solar flux data that is available will not be in any Amateur band. However, the data is taken at nine different frequencies, spread from 245 to 15,400 MHz. A satisfactory way to determine data for in-between frequencies is to plot the current SEC solar flux data against frequency, as shown in an example in Figure 1. Not only the level, but the shape of this curve varies with the day. It is best to plot all nine points to look for any inconsistencies in the data. Also, the use of a "log-log" plot, as shown, prevents the data points from being squeezed on the plot, and emphasizes the shape and slope of the curve. This plot came from a spread-sheet program, but hand plotting is totally satisfactory. Graph paper with the log-log grid is available from some stationary stores, particularly technical suppliers or university book stores.

At this point, we can read from our graph a single number that is the solar flux density corresponding to the time of the measurements and the frequency of interest. Solar flux density is the power received in a given area of the receiving antenna in a 1 Hz bandwidth. As a computational convenience, we will convert the solar flux density, the frequency of operation, and several constants into a number, *i*, the "intensity" measured in Kelvins. As a formula (derived in Appendix I)

$$i = (s/2)\cdot\lambda^2/(4\cdot\pi\cdot k)$$

where s = Solar flux density in Watts per square meter per Hz

λ = Wavelength in meters

k = Boltzmann's constant, $1.38\cdot10^{-23}$ Watt-seconds per Kelvin

As distributed over sources such as the Internet, solar flux density has been scaled up by10^{22}, for which case the units are called "solar flux units," (SFU). Here we add a subscript s when the units are SFU and the equation for *i* becomes

$$i = 0.2882\cdot s_s\cdot\lambda^2 \qquad (1)$$

This equation is plotted in Figure 2 for Amateur frequencies and the normal range of solar flux densities. The equation lends itself to greater precision than the plots, but either method will give the value of *i* that will be needed for calculating *g/t*.

A comment is appropriate on the factor s/2 in the upper equation. This derives from the polarization of the Sun's radiation being random, and only half of this flux level being sensed by any single polarization, either linear or circular. The definition of *s* is the heating value of the flux density while *i*, the intensity, is in terms of the power available at an ideal antenna's terminals. There is no single polarization able to extract the more than half of the energy.

There are other useful noise sources besides the Sun, particularly radio stars and the Moon [5, 8-12]. These can be of great value for measuring the performance of stations with larger antennas, but are too weak to be easily used with small antennas. For that reason, these sources are not covered here. The principles are identical to that shown for the Sun, except that there are no concerns for the angular extent of those sources.

The Quiet Sky - This is our second "known" signal generator. At microwave frequencies, large regions of the sky are devoid of major noise sources. If there were no noise sources where our antenna is pointed, we would be left with only antenna and feed-line losses along with front-end noise, all referred to here as "receiver noise." There are other sources, however:

- The background radiation of the universe. This is only about 3K.
- Radiation from the material in our galaxy, the Milky Way.
- Earth noise picked up by parabolic antenna spillover.
- Earth noise picked up by scattering from antenna feed structures.
- Atmospheric absorption noise.

We should examine these noise sources one at a time to determine if they are a factor in determining our quiet sky noise.

The first source is small in magnitude and unavoidable. So, let's look at radiation from our galaxy. First, one should avoid ever attempting to use an antenna pointed at the galactic plane for a quiet sky. The galactic nucleus is at almost 30 degrees southerly declination, but visible to most stations in the northern hemisphere. The galactic plane is tilted with respect to the Earth's equator and has a declination up to about 65 degrees North. This means that stations can avoid pointing at the galactic plane by keeping the antenna within 20 degrees of the North Star. This point in the sky, obviously limited in view to the Northern Hemisphere, can be found by pointing directly North in azimuth, at an elevation angle equal to the station latitude. Alternatively, the large region around the constellation Cetus-the-whale (declination = 10 degrees S, Right Ascension = 03 hours) is far from the galactic plane and visible from most of the populated Earth. Finding this in az-el coordinates takes some calculations that will not be covered here [14]. Figure 3 shows that at these quiet positions in the sky, the galactic noise is a major factor only below 432 MHz, being about 25 K at 432 MHz and insignificant from 903 MHz and higher. This quantity becomes one component of our total quiet sky noise.

Parabolic antenna spillover occurs when the feed has significant gain past the edges of reflector. To some degree, this spillover is a necessity, as the feed pattern must roll off slowly. Various rules-of-thumb have been offered for maximizing receive performance by trading off antenna gain for reduced spillover, such as that by Barry Malowanchuk, VE4MA [15]. He determined that the illumination at the dish edges should be down 13 to 14 dB from the center for 1989 receiver performance with lower levels needed for better receivers. Paul Wade, W1GHZ, has analyzed numerous dish feeds for their spillover efficiency [16]. These offer a way of estimating the noise contribution of spillover. For instance, the 0.96 λ "Coffee Can" feed in Figure 10 of the referenced article, has about 11% efficiency loss due to spillover for a *f/d* = 0.45 dish. If one assumes that half of this corresponds to side lobes that point toward 290K ground, the noise temperature at the antenna terminals due to this source would be 0.11x0.5x290K or 16K.

Examination of the various figures in the Wade paper shows considerable variation in spillover loss. In addition, the steepness of the spillover curves suggests the importance of under-illuminating the dish for reception purposes, i.e., choose an *f/d* that is on the low side of the maximum gain point. Other antenna types also have unwanted lobes, and can be analyzed for noise in a similar fashion [8].

Antenna feed structures consist of a combination of the electro-magnetics and the mechanical supports for this feed. Center-fed dishes place all of this in the middle where the potential for upsetting the overall radiation pattern is the greatest. This is one argument for the use of an offset feed. A center-fed dish can be tuned to match out some of the intercepted energy that has been reflected back at the feed. But, if the outside of the feed is conductive, this will support current flows that will produce pattern lobes, some portion of which will point towards the Earth, even though the antenna is pointed to the "quiet sky." A conductive support structure can produce the same effect. It is possible to reduce these effects for the supports by the use of absorptive materials, such as wood , that tend to match the impedance of free space. An estimate of the resulting noise energy from the feed structure parallels that of the spillover effects. Again, as an example, if the blockage of the dish was 5% and we assume that half of the re-radiated pattern illuminates the 290K Earth, the resulting noise contribution would be 0.05x0.5x290K or 7K.

Finally, noise from atmospheric absorption losses are principally the result of oxygen and water in the air. The attenuation, in dB, varies directly with the path length through the atmosphere. This, in turn, varies with the cosecant of the elevation angle producing twice the attenuation at a 30 degree elevation angle as that when the antenna is straight up. The following table summarizes the noise temperature (and loss) for several important situations with the antenna pointed straight overhead [17, 18]:

Table 1 - **Zenith Atmospheric Noise Temperature and (Attenuation)**

5.76 GHz and below	2K (0.03 dB)
10.4 GHz, 0% Rel. Humidity	3K (0.04 dB)
10.4 GHz, 100% Rel. Humidity	5K (0.08 dB)
24.1 GHz, 0% Rel. Humidity	4K (0.06 dB)
24.1 GHz, 100% Rel. Humidity	79K (1.1 dB)

Attenuation figures are for a one-way path. An atmospheric physical temperature of 275K is assumed. At frequencies of 5.76 GHz and below, the losses and noise contribution are primarily from oxygen and small. At 24 Ghz, the operating frequency is close enough to the 22 GHz water-vapor absorption peak to make the values very dependent on the relative humidity. At this frequency, there will generally be a significant and difficult to quantify noise contribution from the atmosphere.

Heavy clouds, fog and rain have water droplets that can increase the losses and associated noise. For system performance evaluation, these situations should be avoided if possible. No attempt will be made here to quantify these noise sources, but the references 17 and 18 have curves related to these conditions.

At this point, we should have an estimate of the noise temperature seen by the antenna, called t_Q, (not including receiver noise) when pointing at the quiet sky. The separation between the quiet sky and the "receiver" as used here, places any antenna and feed-line resistive losses as part of the receiver.

Earth Noise - If one can envelope the antenna in a material that is lossy, and at a known temperature, a reference is obtained against which to compare the quiet sky and Sun noise measurements. Since this noise source is large in angular extent, it has a temperature independent of the antenna gain. An approximation to this situation is the Earth, and the material such as forests.

Any lossy material will produce noise at a level determined by the physical temperature and the loss. One concern is that there are reflections from the Earth, so the antenna is also seeing material with a physical temperature other than the Earth's. For instance, if the antenna beam is directed downward towards the ground, the cold sky, or the Sun might reflect back to the antenna. In addition to the noise effects, the reflected wave is able to modify the impedance seen at the antenna terminals. This may alter the gain of the first receiver amplifier, introducing errors into the noise measurements. Ideally, the ground would have an impedance the same as that of free space, producing no reflections and no error sources.

An experiment was conducted to see some of these impedance affects. A 22-element Yagi at 2304 MHz was matched quite carefully when looking straight upward. Then the antenna was pointed towards an extensive oak woods and directly downward. Table 2 summarizes the results:

Table 2 - **Return-Loss Measurements for 22-element Yagi**

Pointed towards sky only	-42 dB
Pointed towards oak woods	-35 dB
Pointed directly downward	-30 dB

Although these mismatches when looking at the Earth are not large, interpretation of these numbers is not simple. One might consider that if a perfect mirror was placed at the ground level, the reflection seen in the mirror when looking downward would be sky plus the antenna itself. Thus the lack of major reflections may not indicate that one is only seeing the lossy Earth; it may be seeing the space in the other direction. Similarly, when noise measurements are being made to small fractions of a dB, changes in input impedance represented by a 30 dB return loss are significant; they are a circle on a Smith chart passing through about 47 and 53 Ohms.

Attenuation measurements made by placing oak boards between two test antennas at 2304 MHz suggest loss of over 1 dB/inch. It is reasonable to expect these losses will increase with moisture content, or with frequency. Thus, the vegetative noise sources at microwave frequencies have much promise. Considerable literature exists on the attenuation effects of trees, as applied to communications between satellites and vehicles [19]. The use of the Earth as a temperature reference has seen much less study and would seem to be a fruitful and useful area for amateur researchers. The satellite studies, such as reference [19] show considerable attenuation for forested areas, but these studies do not usually address the reflections from the forest. The use of mineral matter for a temperature reference seems to have had no study.

If the antenna-terminal impedance changes with antenna direction, the receiver first stage preamplifier gain will change as well. This gain change will appear the same as a change in noise level. The amount of gain change depends on both the particular impedance and the stability of the amplifier. Measuring this gain change requires care and the proper test equipment. As an alternative, ferrite isolators can be placed ahead of the receiver. These devices reduce the changes in impedance seen by the amplifier. Two isolators can be placed in cascade to further reduce these effects. Some noise will be introduced by the loss in the isolator. This can be evaluated by measuring the insertion loss of the isolator, L_i (in dB), and knowing the physical temperature, t_i (in Kelvins). The noise temperature added to the receiver temperature will be $t_i \cdot (10^{0.1 \cdot L_i}-1)$. This can be subtracted after the system noise temperature is measured, assuming that the isolator(s) is not left as part of the receiver.

As an alternative to Earth measurements, a resistive load is quite attractive. The physical temperature is easy to determine and the impedance match can be very good. The negative side to the use of such a load is the need to modify the system being measured by inserting the load. Again the use of ferrite isolators can be of benefit here.

One way, or another, at this point we should have an estimate of the noise temperature seen by the antenna, called t_E, (not including receiver noise) when pointing at the Earth. If measurements are not available, assuming a value for t_E in the 220 to 250K range appears to be reasonable.

Measuring Power Ratios - The estimation of performance requires measuring the ratio between two noise powers. A number of approaches are available to keep the measurement radio operating in its linear region, to do power averaging and to accurately produce a power value [3, 4, 12, 20]. Part of what must be dealt with is a trend toward commercial radios that cannot have their AGC disabled. If using such a radio, be sure that backing off the RF gain is an adequate means for disabling the AGC. In addition, the meter used to measure the power levels must have accuracy over a wide dynamic range. A good quality RMS AC digital voltmeter can provide excellent accuracy.

Another approach is to use the very high linearity of delta/sigma analog-to-digital converters. These are normally the type used with the audio ports of computers. To make full use of these requires software, of course. A single package that can be specifically configured for noise-measurement purpose is the DSP-10 radio [21]. That 2-meter transceiver has an I-F crystal filter with a bandwidth of about 12 kHz. The entire bandwidth is sampled at a 48 kHz rate with true power calculations (I^2+Q^2) for each sample. These powers are averaged to any amount desired and displayed to 0.001 dB resolution. An accuracy adequate to support the resolution is maintained by the A/D converter, even for large y-values. The 12 kHz bandwidth is not as wide as one might want for fast measurements, but it does make it easy to find measurement frequencies that do not have interfering stations. To obtain a noise sample with a standard deviation of 0.01 dB takes about 16 seconds. This is usually adequate accuracy for amateur purposes. In addition to making the noise measurements, the DSP-10 can display the azimuth and elevation coordinates of the Sun.

Performance Measure, *g/t* - The best receiving performance is obtained by maximizing the ratio of the antenna gain divided by the system noise temperature, usually called *g/t*. This says that increasing the gain is only beneficial if it happens faster than the noise increase from the resulting side/back lobes. Usually the *g/t* ratio is expressed in dB as $10 \cdot \log_{10}(g/t)$, which often carries the nick-name for units of dB/K. Again, to avoid dB confusion, we will not refer to a quantity "G/T" since after conversion to dB, the *g/t* ratio is taken by subtraction of dB quantities, rather than by division. That is,

$$10 \cdot \log10(g/t) = 10 \cdot \log10(g) - 10 \cdot \log10(t) = G - T \text{ and not G/T.}$$

If the solar flux levels are known, the value of *g/t* can be determined from the measurement of two noise powers, the quiet sky and the Sun. The ratio of the two noise powers is often used as a substitute measure for *g/t*, but as is often pointed out, this is ambiguous unless the solar flux is also specified. Much better is to estimate *g/t*.

If we have followed through the above process of finding the solar intensity, *i*, only two noise power measurements are needed to determine *g/t*. The noise measurements with the antenna pointed at the quiet sky, and with the antenna pointed directly at the Sun tell the whole story. The formula is (see Appendix I)

$$g/t = (y_{SQ}-1) / i \qquad (2)$$

where

$y_{SQ} = n_S/n_Q$

n_Q = noise power received with the antenna pointed at the quiet sky

n_S = noise power received with the antenna pointed at the Sun

The noise measurements are not calibrated in any absolute sense. Since they both have the same (implied) reference, the reference cancels out when the ratio y_{SQ} is taken. This normally occurs by measuring the quiet sky on a power meter and moving the antenna to the Sun. By observing the number of dB of power increase we have measured Y_{SQ}, in dB. This is converted to y_{SQ}, the ratio, when we apply

$$y_{SQ} = 10^{(0.1 \cdot Y_{SQ})}$$

which is the reverse formula from that used to convert to dB. Alternatively, one can get *g/t* to reasonable accuracy by applying the graphs of figure 4.

Receiver and System Noise Temperatures - The quantity *g/t* is fine as a measure of receiving performance. It lacks, however, in showing how the performance divides between the antenna gain and the system noise temperature, *t*. This becomes important if we are wondering where to apply our efforts to improve receiving performance. Additionally, it does not show transmitting performance, since that is independent of system noise temperature.

As discussed above, when the antenna is pointed at a lossy medium that fully encompasses the antenna beam, we can deduce the resulting noise power seen at the antenna "terminals." This measurement, combined with the quiet sky noise power measurement provides an estimate of the receiver noise temperature. We again take the ratio of these two measurements, y_{EQ}, where

$y_{EQ} = n_E/n_Q$
n_E = noise power received with the antenna pointed at the Earth
n_Q = the quiet sky measurement again.

When the antenna is pointed at the quiet sky, the noise power is the sum of that due to the receiver and that due to the quiet sky (and undesired lobes, as discussed previously). When the antenna is pointed at the Earth, the noise power is the sum of that due to the receiver and that due to the Earth, as discussed above. Expressed as an equation:

$$y_{EQ} = (t_E + t_R)/(t_Q + t_R) \qquad (3)$$

where

t_R = the receiver noise temperature to be determined
t_E = the assumed temperature for the Earth.
t_Q= the assumed temperature for the Earth

We have rather casually switched the measure of noise power from the Watts to Kelvins, but these two quantities only differ by a constant. Whenever ratios are taken, the constant cancels out, allowing this change in units. If we solve for t_R:

$$t_R = (t_E - t_Q \cdot y_{EQ})/(y_{EQ} - 1) \qquad (4)$$

This is an estimate of the receiver noise, expressed as a noise temperature, not including external noise sources. This equation is plotted in the six graphs of figure 5. Separate pairs of graphs cover three different Earth temperatures, 220, 255 and 290K. Other values can be interpolated or calculated directly from the equation for t_R.

If we want the system noise temperature, t, we would add on the quiet sky noise temperature

$$t = t_R + t_Q \qquad (5)$$

If the temperature standard is not the Earth, but rather a matched load of known physical temperature, the procedure still works as described, but the load temperature is substituted for the Earth temperature.

Antenna Gain - Now that we have an estimate of the system noise temperature and g/t, the antenna gain is merely

$$g = t \cdot g/t \qquad (6)$$

This is a ratio relative to the gain of an isotropic radiator, and the more conventional dB value, G, is

$$G = 10 \cdot log_{10}(g)$$

This is a direct component of the station performance when transmitting.

Error Estimates - To this point we have been reviewing the general concept of Sun/Earth noise measurements. A question remains of the practicality of doing such measurements, given that the data is not perfect. Appendix I includes the equations for errors in the estimation of g/t, t_R and g. These are very suitable for calculations in a small program, and such a program is in the appendix. Alternatively, the graphs in Figures 4 and 5 are able to give some insight into these errors, offering a feel for the sources of errors, rather than just presenting numbers.

For example, the measurements of the y values are never perfect. These involve the ratio of two different noise measurements. For example, we might examine the effect of an error in Y_{SQ} on the estimation of g/t.

The two graphs in figure 4 shows that for Y_{SQ} greater than about 4 dB, errors track dB for dB. Smaller values of Y_{SQ} produce larger errors; at Y_{SQ} = 1dB, the spacing between the graphs for Y_{SQ}=1 and Y_{SQ}=2 is about 3-1/2 times what it is for larger values of Y_{SQ}. This tells us that an error of 0.1 dB in Y_{SQ} will produce an error of about 3.5 x 0.1 or .35 dB. We will see that this same conclusion comes from error calculations.

Similar insights can come from examining the receiver noise temperature graphs of figure 5.

Returning to the actual error calculations, Appendix I derives equations for what are referred to as "normalized sensitivities." The notion is that a parameter, such as *g/t*, can be expressed in terms of the basic measurements, such as solar flux and the y_{SQ} power ratio. We can then consider a small fractional error in each of the measurement and translate this into a fractional error in *g/t*. Fractional errors are used for exactly the same reason that we use dB notation. Our interest is often in not in the actual error value, but rather what fraction it is of the parameter. For instance, if *g/t* is 5 and we have an error of 1, this error is quite large. But, if *g/t* was 20, an error of 1 has only 1/4th the interest. So the normalized sensitivities are used here exclusively, and have the form

$$\frac{\text{Error in a Parameter}}{\text{Parameter Value}} = \text{Normalized Sensitivity} \times \frac{\text{Error in the Measurement}}{\text{Measurement Value}}$$

This formulation makes a normalized sensitivity of 1.0 as "average sensitivity," and smaller values as insensitive. Values larger than 1.0 are the ones more likely to be the major sources of error. An additional advantage is that the same normalized sensitivity will relate small dB values in the same way that it does for small errors. For this reason, when considering the normalized sensitivities, small errors and dB values get treated casually, but of course, these two should never be mixed. For instance, if we go to figure 6A, the normalized sensitivity of *g/t* to y_{SQ}, we see a value of about 1.65 for y_{SQ} = 4 dB. This means that an error of 0.1 dB in y_{SQ} would produce an error in *g/t* of about 0.165 dB. However, it also means that if we have a fractional error of 0.02 (2%) in y_{SQ}, the fractional error in *g/t* would be about 1.65 times this amount or 0.033 (3.3%).

Figure 6A through 6D are graphs of the normalized sensitivities for estimating *g/t* and t_R. These are for selected values of parameters, as indicated on the plots. To present graphs for all the possible range of measurement values would require an excessive amount of paper. The graphs shown will evaluate errors for many common systems. If this is not adequate, the equations are given in the appendix for all the sensitivities, and the Basic program calculates these as well.

As a result of the error analysis, we will end up with multiple errors due to different measurements. How do we combine these components to produce a single error estimate? These are statistical errors for which we do not know an actual value nor even the sign of the error. If we knew the actual error value, we would remove it! We often use a phrase such as, "we have an error, or uncertainty, of 0.1 dB." What we usually mean is that the error is most likely zero (the statistical mean value) and varies about that in some way described by, perhaps, a standard deviation (sigma) of 0.1 dB. Or maybe the 0.1 dB is three sigmas, or some other measure. So what happens when we try to add two or more errors together when they are statistical? If the number of errors added together is small, the answer is not obvious. In the appendix, it is shown that addition of the individual errors is appropriate, if we only knew their values. It is not correct to add the sigma's of the error values, and to do so results in a conservative, or "worst case" error estimate (one alternate is to root-sum-square, or RSS the sigma's). On the other hand, if the number of variables being added is only two or three, and we have only one or two measurements, it may be appropriate to be safe and add the error values.

What utility do these error measurements have? When comparing two systems, or a single system at two different times, it keeps us from confusing real differences from those due to measurement errors. System A measures a *g/t* of 12 dB/K and system B measures 13 dB/K. One might conclude that B was better, given no other data. But, if both measurements resulted in an error of 1 dB, we could say that system A was 11 to 13 and B was 12 to 14 dB/K. Now we know that system A might really be the better of the two. This might be good to know if we were about to take system A apart to find out what was wrong! This example also illustrates the utility of identifying a need to improve the measurement accuracy. For this case, the accuracy of 1 dB doesn't seem adequate!

An Example - Assume that a 1-meter diameter parabolic antenna is being measured in late morning on 27 June 2003, so that the solar flux data of figure 1 is accurate. In general, we would have gone to the Internet and obtained the data to plot a curve appropriate to that time. At our operating frequency of 10368 MHz, the flux level is taken from the curve as 315 SFU. Using the 10.4 GHz graph of figure 2 (or equation 1), the value of i is found to be 0.0760 K.

The measurements found the difference in noise between the Sun and the quiet sky around the North star to be Y_{SQ} = 5.94 dB. This was done by pointing the antenna at the quiet sky around the North Star, measuring the noise power coming from the receiver in relative dB, and then moving the antenna to the Sun and making a second noise power measurement. Y_{SQ} is the difference in the two noise power dB levels. The Earth noise was measured similarly as Y_{EQ} = 4.07 dB above the same quiet sky. Several measurements were made and an accuracy of 0.1 dB was determined for these Y ratios.

From the first of these measurements, along with our value for i, we can estimate g/t by using graphs of figures 4 to be:

```
g/t = 15.9 dB/K
```

The graph does not include a line for exactly Y_{SQ} = 5.94, so we must interpolate by moving very slightly away from the 6.0 line, towards the 5.0 line. Alternatively, this could have been found with a calculator using equation 2, or the equivalent calculation in the Basic[1] program of Appendix I.

The g/t number is very useful for describing our receiving performance. Next we can examine the sensitivity of g/t to errors in Y_{SQ} and i. For this we can use the graph of Figure 6A (or equations A1-6 and A1-7), noting that the sensitivity to i is always -1:

```
Normalized Sensitivity to ySQ = 1.34
Normalized Sensitivity to i = -1.00
```

Subscripts have been left as full size characters, as was necessary in the Basic program, but `ySQ` is really y_{SQ}. The performance numbers give a good measure of the antenna and receiver performance. It is interesting to look at the sensitivities, to see how accurate these numbers might be. For g/t , the sensitivity to y_{SQ} is 1.34. This means that our 0.1 dB accuracy in Y factors will be about 0.134 dB in g/t. This is not anything like a .001 dB precision activity, so we can round things off and say the accuracy in g/t is still about 0.13 or even 0.1 dB. The solar flux numbers are not perfect, either. Part of this is from the basic measurement accuracy and part from the interpolation of the flux vs. frequency curve. An error of 5% in flux produces an error of 5% in i and from the sensitivity, we can see this produces an error in g/t of -5%. The minus sign is not important since we don't know the direction for the flux error that started this. Since we have g/t in dB, we should convert the possible error to dB as well, by $10 \cdot \log10[(1.0+.05) / 1] = 0.2$ dB. As a worst case estimate we might add the two errors and conclude that the g/t is 15.9 ± 0.3 dB.

The g/t estimate has not required any information about the quiet-sky noise temperature and neither has it used the Earth noise measurement. Let's proceed to estimate the receiver noise temperature by including these quantities. We can first estimate the quiet sky noise. A t_Q = 30 K quiet sky is based on:

```
Background radiation                 3 K
Galactic radiation                   0 K
Noise from antenna spillover        16 K
Noise from feed structure scattering  7 K
Atmospheric absorption noise         4 K
```

[1] Many of the calculations in the example are produced from the Basic program.

```
        Total Quiet Sky                  30 K
```

The Earth noise is measured by pointing the antenna into a wooded area at a physical temperature of 68F or in Celsius or absolute scales 20 C and 293 K. We know that the match of this "Earth" target is not perfect and some of the observed noise is reflected from much colder space. Ideally, we would measure this temperature by comparing against a reference load, but lacking this data we will estimate $t_E = 250$ K. We can now estimate the receiver noise temperature using Figure 5E (or equation 4):

```
tR = 112 K
```

Sensitivities come from figures 6B, 6C and 6D:

```
Normalized Sensitivity to tE = 1.44
Normalized Sensitivity to tQ = -0.44
Normalized Sensitivity to yEQ = -2.61
```

The sensitivity to y_{SQ} is 2.6 and enough larger than one to warrant special consideration. Our 0.1dB accuracy in Y factors produces about 0.26 dB in t_R. To see what this means, we will translate into an error, in t_R, into temperature units by changing 0.26 dB into a power ratio of 1.062, suggesting a range of t_R of 112 / 1.062 to 112 x 1.062, or 105 to 119 K. Perhaps of greater concern is the sensitivity to t_E, since we were not very confident in our original estimate of the Earth temperature. If we have an error of 30 K in our 250 K for t_E, this is a fractional error of 30 / 250 = 0.12. The fractional error in t_R will be 1.44 x 0.12 = 0.17 and in Kelvins, the error in t_R would be 0.17 x 112 = 19 K.

Our uncertainty in t_Q might center on the spillover contribution, which might be ±5 K. Since t_Q is small, this raises the level of this contributor, where the fractional error is 5 / 30 = 0.17. This contributes an error to t_R of 0.17 x 112 = 19 K.

We now have three independent errors of 7, 19 and 19 K. The worst case situation of adding the errors gives 45 K. If we had probability distributions for the errors, we might be formal about producing an actual distribution of the errors. This is probably not justified in terms of our actual knowledge of the measurement errors. What is worthwhile is to RSS the errors and use this as our "less conservative" error value. This would have an error of SQRT($7^2 + 19^2 + 19^2$) = 28 K. On this basis, our uncertainty now covers roughly 112-28 to 112+28 or 84 to 140 K. This relatively large uncertainty does not mean the method is not useful, but rather suggests a need to reduce the uncertainty in t_E. In fact, expressed as noise figures, the uncertainty is 1.4 ± 0.3 dB and not a great deal larger than the accuracy of noise figure meters!

Finally, we directly calculate the system noise temperature and the antenna gain

```
t = tR + tQ = 142 K   (system noise temperature)

g = t x g/t = 5524, or 37.4 dB  (antenna power gain)
Normalized Sensitivity to ySQ = 1.34
Normalized Sensitivity to yEQ = -1.64
Normalized Sensitivity to i = -1.00
Normalized Sensitivity to tQ = -0.136
Normalized Sensitivity to tE = 1.14
```

The sensitivities for the system noise temperature are the same as for the receiver noise temperature and not repeated. The normalized sensitivities for the antenna gain, *g*, have five components (treating all as being positive):

```
 ySQ:  0.1 dB   or fraction=.023   Fraction of ySQ=1.34x.023 = .031
 yEQ:  0.1 dB   or fraction=.023   Fraction of ySQ=1.64x.023 = .038
   i:  5%       or fraction=.05    Fraction of ySQ=1.00x.05  = .05
  tQ:  5K/30K   or fraction=.167   Fraction of ySQ=.136x.167 = .023
  tE:  30K/250K or fraction=.125   Fraction of ySQ=1.14x.12  = .14
```

It is apparent that the error in the determination of antenna gain for this example is dominated by our uncertainty in the Earth temperature. The error due to this factor alone is about $10\cdot\log_{10}(1.14) = 0.57$ dB. If we worst-case add all the error fractions, we have a fraction sum of 0.282 or 1.1 dB. If we RSS the errors we have 0.157 as a fraction or 0.64 dB. This is telling us that exact number we cannot be precise about, but we should allow for an error in gain around "one dB." Equally important, if we want to improve this accuracy we should first improve our knowledge of the Earth temperature.

Again, this is only an example. The numerical values may be valid for the system studied, but they will to be different for each individual case.

Conclusions - The graphs of figures 4 and 5 apply the three-noise measurement scheme for determining system performance parameters, *g/t*, *t* and *g*. These are suitable for determining the performance, but perhaps a better method is the simple Basic program of Appendix I. The program quickly shows the specific answers for the measurements taken. The graphs are best for gaining insight about the method. One can see the way in which the performance parameters vary as the measurements vary.

The program calculates the sensitivities for *g/t*, t_R (or *t*) and *g* to errors in the various input variables. A normalized sensitivity of 1 indicates that a small percentage error (or small dB error) in an input variable will result in the same percentage error (or dB error) in the performance parameter. Other values of the normalized sensitivity scale the parameters accordingly. After estimating the performance parameters, the sensitivities should be used to find the general order of magnitude of the errors. It is not generally necessary to make a statistical combination of the sensitivities.

In order to compare system performance between different times, as well as to compare different systems, it is necessary to have obtained a value for the solar flux and the temperatures of the quiet sky and Earth. This requires a bit of extra effort, but allows a much more useful estimate than Sun-noise ratios. The step of computing *g/t*, t_R (or *t*) and *g* provides the most useful form of the answers. Finally, if the magnitude of the errors are estimated, it is possible to place statistical bounds on the performance parameters.

Finally, here are some general rules for consideration:

- Whenever taking Sun noise measurements, also take Earth noise measurements.
- Obtain current solar flux numbers and interpolate for the frequency used.
- Convert Sun noise into *g/t* values and use this as a primary measure of receiving performance.
- Calculate the receiver and system noise temperatures.
- Calculate the antenna gain, as it is the single parameter that relates to transmitter performance.
- Try to estimate the errors in the measurements and parameters.
- Keep the measurements in a notebook where they can be found when questions arise!

Acknowledgement - Many thanks to Ernie Manly, W7LHL, for sharing his wisdom in the art of microwave measurement and for his review of this paper, that was most helpful.

Notes and References:

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2 - Melvin B. Graves, WR0I, "Estimating Sun Noise at Various Frequencies, Based on the 10.5 cm Flux Reported by WWV," *Proceedings of Microwave Update '94*, ARRL, pp 125-131. Covers estimation of solar flux data from the commonly used 10.5 cm data. Up-to-date solar flux for a variety of microwave wavelengths is now available over the Internet and, for many operators, solves this problem.

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4 - Rick Fogle, WA5TNY, and Les Whitaker, W7CNK, "Calculated Values for Sun and Moon Noise for Common Size Microwave Antennas," Proceedings of Microwave Update '89, ARRL, pp 173-178. Sun noise predictions.
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11 - William Curry, Jr., W5CQ/W4RXY, "Antenna-Performance Measurements using Celestial Sources," Ham Radio, May 1979, pp 75-79. Covers finding G/T from the Sun, Moon and radio stars.
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14 - Astronomers normally use a coordinate system of declination and right ascension (RA). These are the quantities needed to describe a position for astronomy books. RA (in hours) is converted to the navigator's term "sidereal hour angle," (SHA), in degrees, by SHA = 360 - (360 / 24) * RA. This is now in the the terms for use with the *Nautical Almanac*, as described in chapter 10 of the *UHF/Microwave Experimenters Manual*, ARRL, 1990.
15 - Barry Malowanchuk, VE4MA, "Selection of an Optimum Dish Feed," *Proceedings of the 23rd Conference of the Central States VHF Society*, ARRL, 1989, pp 35-43.
16 - Paul Wade, W1GHZ, Parabolic Dish Feeds - Performance Analysis," *Proceedings of Microwave Update '97*, ARRL, pp 6-34. This paper, along with others that the same author has published give curves for spillover and feed blockage, both of which tie closely with antenna side and back lobes.
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20 - Bob Atkins, KA1GT, "Noise Temperature, Antenna Temperature and Sun Noise," *The UHF/Microwave Experimenter's Manual*, ARRL, 1990, Chapter 7, pp 58-59. Estimation of receiver noise temperature using the Sun noise and an assumed antenna gain. Be aware that there is an error in Eq. 3, that the term (NF/10) should be a power of 10, not multiplied by 10 (Eq 6 on page 7-55 is correct).
21 - Bob Larkin, W7PUA, "The DSP-10: An All-Mode 2-Meter Transceiver Using a DSP I-F and PC-Controlled Front Panel," *QST*, Sept., Oct. and Nov. 1999. http://www.proaxis.com/~boblark/dsp10.htm is the home page for the DSP-10 project, and has the software available at no-cost. Referenced from that page are numerous related links. All versions of the software have the high resolution noise measurements in the FM mode, but version 3.06 and later have the provision for unlimited averaging and should be used for the noise measurements.

Appendix I - Equation Derivation, Error Analysis and Basic Program

The basic equations for Sun noise are presented in reference [6] and elsewhere. The assumption is that the Sun is smaller in angular extent than the antenna beam doing the measuring. This is valid except for very large antennas, or very high frequencies, as the Sun's diameter is about 1/2 degree. The solar flux arrives at the Earth's surface with a uniform illumination of s solar flux units (SFU) with an SFU=$1\cdot 10^{-22}$ Watt-seconds / square-meter. If we characterize the antenna as an energy collector with an effective area of a_e, the intercepted energy for any single antenna polarization will be $(s/2)\cdot a_e$ Watt-seconds. To convert this to an effective temperature, which is the black-body temperature that would produce the same noise energy, we divide by Boltzman's constant, $k=1.3806\cdot 10^{-23}$ Watt-second/Kelvin. If we call the equivalent Sun temperature t_{SN}, we have

$$t_{SN}, = (s/2)\cdot a_e / k \qquad \text{(A1-1)}$$

The gain of an antenna, g, is a more conventional description of performance, and its effective area, A_e, is given by

$$g = 4\pi\, a_e / \lambda^2 \qquad \text{(A1-2)}$$

Eliminating a_e, we have

$$t_{SN}, = g\, s\, \lambda^2 / 8\pi\ k$$

For convenience, s is converted into an "intensity" i, in Kelvins

$$i = s\, \lambda^2 / 8\pi\ k$$

so that

$$t_{SN} = g\cdot i \qquad \text{(A1-3)}$$

The measurement of Sun noise consists of determining the ratio of the noise powers for the antenna pointed at the Sun and at the quiet sky. We will describe all these powers in terms of equivalent temperatures, the usual convention, keeping in mind that these relate to actual received powers by a constant. For the Sun case we receive the Sun, plus the quiet sky, plus the receiver noise. For the quiet sky, we receive the latter two quantities only. Thus

$$y_{SQ} = (t_{SN} + t_Q + t_R) / (t_Q + t_R) \qquad \text{(A1-4)}$$

which can be expressed in terms of the system noise temperature $t = t_Q + t_R$ as

$$y_{SQ} = (t_{SN} + t) / t$$

Substituting for t_{SN} from A1-3, and solving for g/t we have the fundamental Sun noise equation used in figure 4 of the text

$$g / t = (y_{SQ} - 1) / i \qquad \text{(A1-5)}$$

The y measurements and values of i will have errors. These can be evaluated by a standard technique of partial derivatives. For small errors, we evaluate the error in g/t, called $\Delta(g/t)$ as the sum of two terms that relate to each of the quantities that may have errors, y_{SQ} and i. The formulation is

$$\Delta(g/t) = \frac{\partial(g/t)}{\partial y_{SQ}}\cdot \Delta y_{SQ} + \frac{\partial(g/t)}{\partial i}\cdot \Delta i$$

To those not familiar with this, it may have elements of gibberish! However, it is a precise statement that roughly translates in words to, "an error in g/t is the sum of errors in both variables, y_{SQ} and i, multiplied by their sensitivities. The partial derivative means that when we evaluate the sensitivity of g/t to y_{SQ} we can treat i as though it was a constant, and when evaluating the sensitivity of g/t to i we can treat y_{SQ} as though it was a constant. A subtlety of the equation is that since it is a sum of terms, we can consider the errors associated with y_{SQ} and i separately, a major convenience. The actual evaluation of the partial derivatives is by differential calculus, which for this case results in an equation:

$$\Delta(g/t)=\frac{1}{i}\Delta y_{SQ}-\frac{y_{SQ}-1}{i^2}\Delta i$$

In evaluating this equation, remember that all variables are in their basic units and not dB.

In all that follows, we will normalize the sensitivities by dividing by the quantity with an error. This has the effect of always talking about "percentage" errors, and since we are assuming small errors, it is a good approximation to say that these normalized sensitivities are also the multiplier for dB of error. The independence of the (small) error contributions is used here by assuming, for example, that Δi is zero when evaluating the sensitivity to y_{SQ}. For g/t, the two error equations are (A1-6 and 7):

$$\frac{\Delta(g/t)}{g/t}=\frac{y_{SQ}}{y_{SQ}-1}\cdot\frac{\Delta y_{SQ}}{y_{SQ}}$$

$$\frac{\Delta(g/t)}{g/t}=-\frac{\Delta i}{i}$$

In the first of these equations, it appears that one might cancel the terms y_{SQ}, but this would remove the normalization that allows us to see the ratios of percentages and dB's.

Moving to the estimation of receiver noise temperature, t_R, by use of the Earth and Quiet Sky, we won't show the detail that was presented for *g/t*, but the method is identical. The resulting equations in normalized form are (A1-8, 9 and 10):

$$\frac{\Delta t_R}{t_R}=\frac{t_E}{(t_E-t_Q\cdot y_{EQ})}\cdot\frac{\Delta t_E}{t_E}$$

$$\frac{\Delta t_R}{t_R}=\frac{-t_Q\cdot y_{EQ}}{(t_E-t_Q\cdot y_{EQ})}\cdot\frac{\Delta t_Q}{t_Q}$$

$$\frac{\Delta t_R}{t_R}=\frac{y_{EQ}\cdot(t_Q-t_E)}{(t_E-t_Q\cdot y_{EQ})\cdot(y_{EQ}-1)}\cdot\frac{\Delta y_{EQ}}{y_{EQ}}$$

Finally, for the case of determining antenna gain, the calculation of g depends on y_{SQ}, i, y_{EQ}, t_Q and t_E, but the sensitivities are the same as those of *g/t* for the first two variables and will not be repeated here. For the other three variables (A1-11, 12):

$$\frac{\Delta g}{g}=\frac{t_Q(1-y_{SQ})}{g\cdot i\cdot(y_{EQ}-1)}\cdot\frac{\Delta t_Q}{t_Q}$$

$$\frac{\Delta g}{g}=\frac{t_E(y_{SQ}-1)}{g\cdot i\cdot(y_{EQ}-1)}\cdot\frac{\Delta t_E}{t_E}$$

$$\frac{\Delta g}{g}=\frac{y_{EQ}(y_{SQ}-1)\cdot(t_Q-t_E)}{g\cdot i\cdot(y_{EQ}-1)^2}\cdot\frac{\Delta y_{EQ}}{y_{EQ}}$$

where g has been written in fundamental terms (A1-14):

$$g = \frac{(y_{SQ} - 1) \cdot (t_Q y_{EQ} - t_Q + t_E)}{i \cdot (y_{EQ} - 1)}$$

The equations for *g/t*, *t*, and *g*, along with all sensitivities can be evaluated from the following Basic program. If Earth noise measurements are not available, the program can still be used, but the only valid output is *g/t*.

```
REM SQE1.bas  Sun, Quiet Sky and Earth noise calculations.
REM  (c) Bob Larkin, W7PUA, 2003

PRINT "SQE1.BAS Program - de W7PUA, Rev 1.0, 28 June 03"
PRINT "Sun, Quiet Sky & Earth Noise Calculations"
INPUT "Quiet sky temperature, tQ, Kelvins"; tQ
INPUT "Earth temperature, tE, Kelvins "; tE
INPUT "Sun intensity, i, Kelvins "; i
INPUT "Sun/Quiet power ratio, dB "; YSQ
INPUT "Earth/Quiet power ratio, dB "; YEQ

yS = 10 ^ (.1 * YSQ)
yE = 10 ^ (.1 * YEQ)
gt = (yS - 1) / i
PRINT USING "g/t =###.#"; 4.342945 * LOG(gt);
PRINT USING " 'dB/K', or as a ratio g/t =######.#"; gt
sensgtysq = yS / (yS - 1)
REM Sensitivity of g to i simplifies to -1, a constant
PRINT USING "g/t Error Sensitivities: ##.## to ySQ"; sensgtysq;
PRINT " and -1.00 to i"
tR = (tE - tQ * yE) / (yE - 1)
PRINT USING "Receiver noise temperature, tR =####.# Kelvins"; tR
PRINT USING "tR Error Sensitivities: ##.## to tE,"; tE / (tE - tQ * yE);
PRINT USING "  ###.## to tQ,"; -tQ * yE / (tE - tQ * yE);
PRINT USING "  ###.## to yEQ"; yE * (tQ - tE) / ((tE - tQ * yE) * (yE - 1))
PRINT USING "System noise temperature =####.# Kelvins"; tR + tQ
g = gt * (tR + tQ)
PRINT USING "Antenna gain, G = ##.# dB"; 4.342945 * LOG(g)
REM Sensitivities for antenna gain
REM Sensitivity of g to both i and ySQ is the same as for g/t
dgdtq = (1 - yS) / ((yE - 1) * i)
senstq = dgdtq * tQ / g
dgdte = (yS - 1) / ((yE - 1) * i)
senste = dgdte * tE / g
dgdyeq = (yS - 1) * (tQ - tE) / (i * (yE - 1) * (yE - 1))
sensyeq = dgdyeq * yE / g
PRINT "g Error Sensitivities"
A$ = "   ##.## to ySQ,  ##.## to yEQ,  -1.00 to i,  ##.### to tQ, ##.## to tE"
PRINT USING A$; sensgtysq; sensyeq; senstq; senste
```

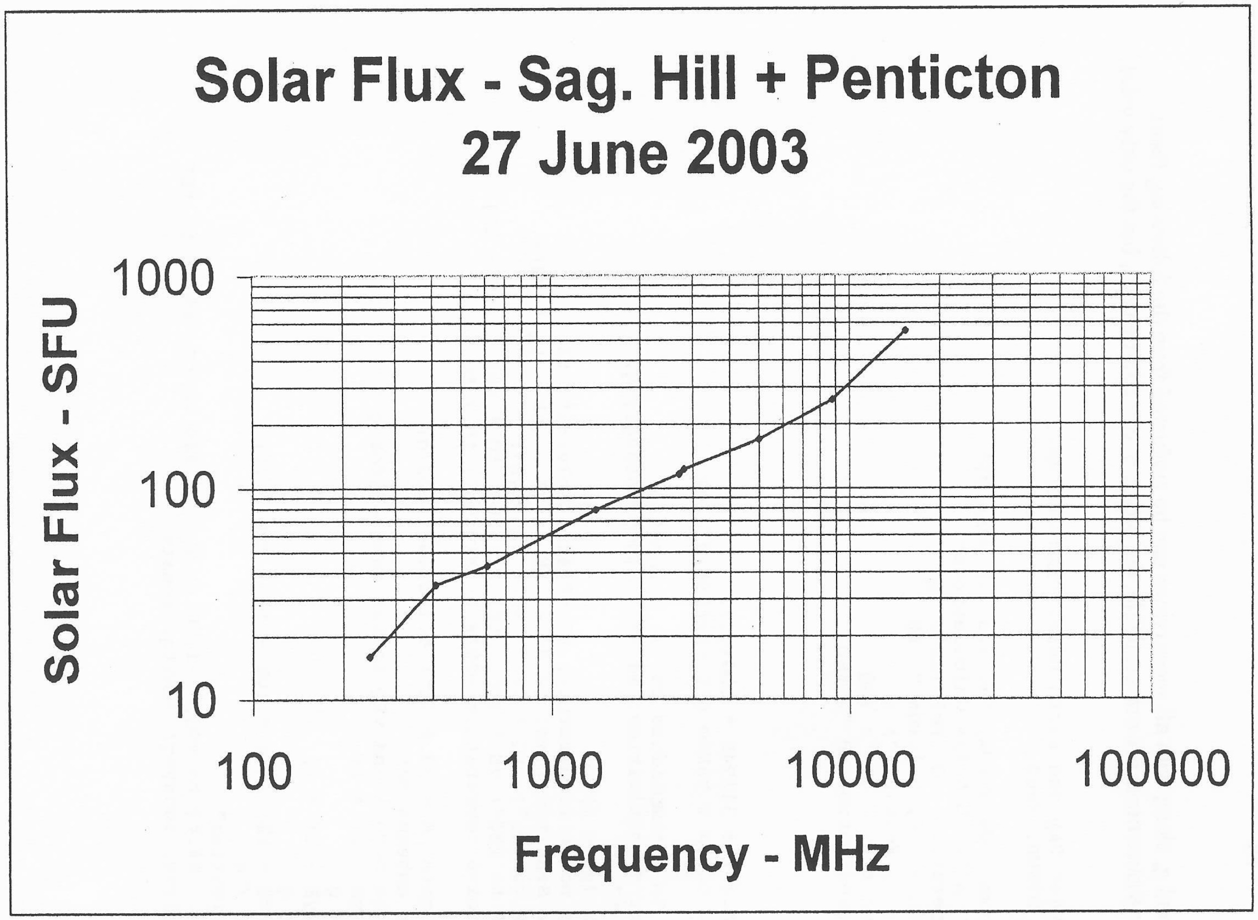


Figure 1 - The individual data points are the measured solar flux density as posted to the Internet. The point at 2800 MHz (the right-hand point of the double point) comes from the Canadian Penticton, British Columbia, observatory; these are the measurements used by WWV. The remainder of the points are from the Sagamore Hill, MA, observatory. By plotting the points it is easy to interpolate the flux values for non-measurement frequencies, such as the amateur bands. Both the values of the solar flux and the variations with frequency change from day-to-day and the use of a generalized curve is less accurate than a specific curve showing the current data.

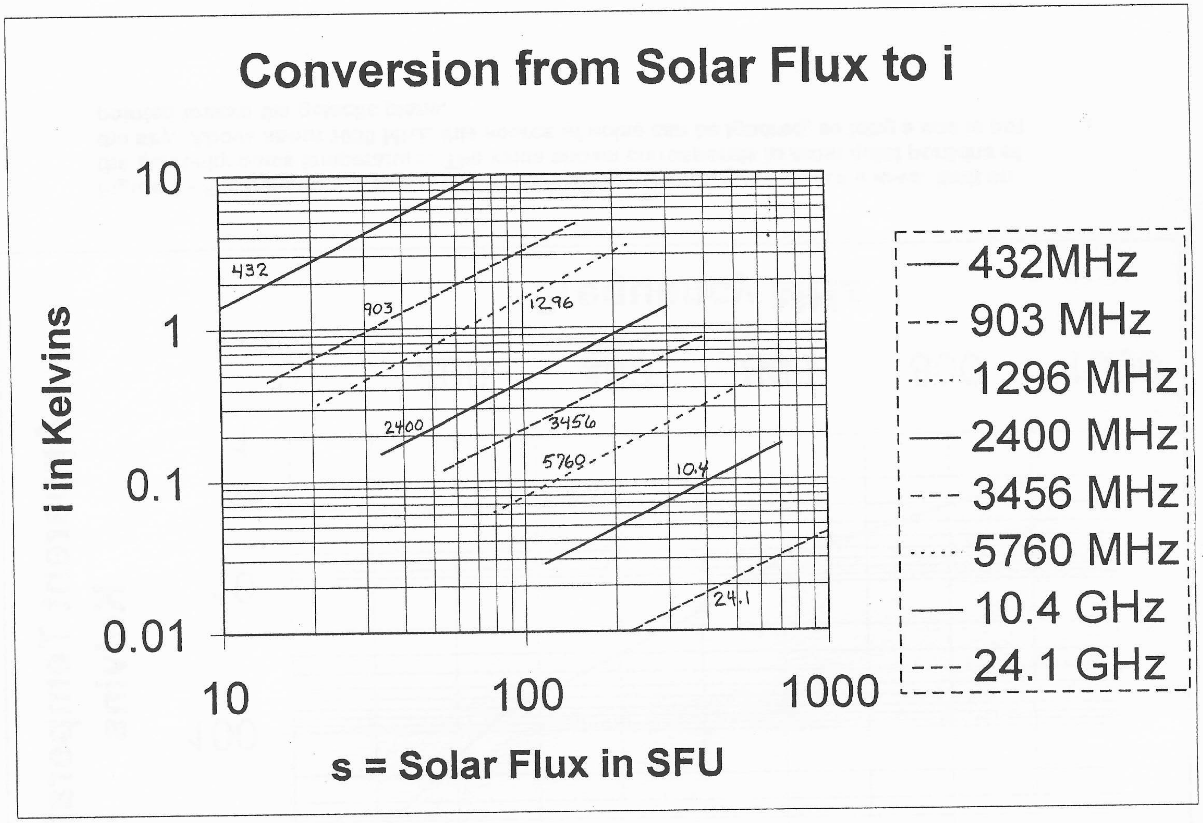


Figure 2 - The quantity *i* is derived directly from the solar flux and the frequency of operation. This is a computational convenience in determining g/t and the other performance parameters. To use this figure, select the graph line corresponding to the frequency of interest, obtain the solar flux density from figure 1 and enter it to the horizontal axis, and read the *i* value off the vertical axis. This graph is from equation 1 which can be used for greater precision.

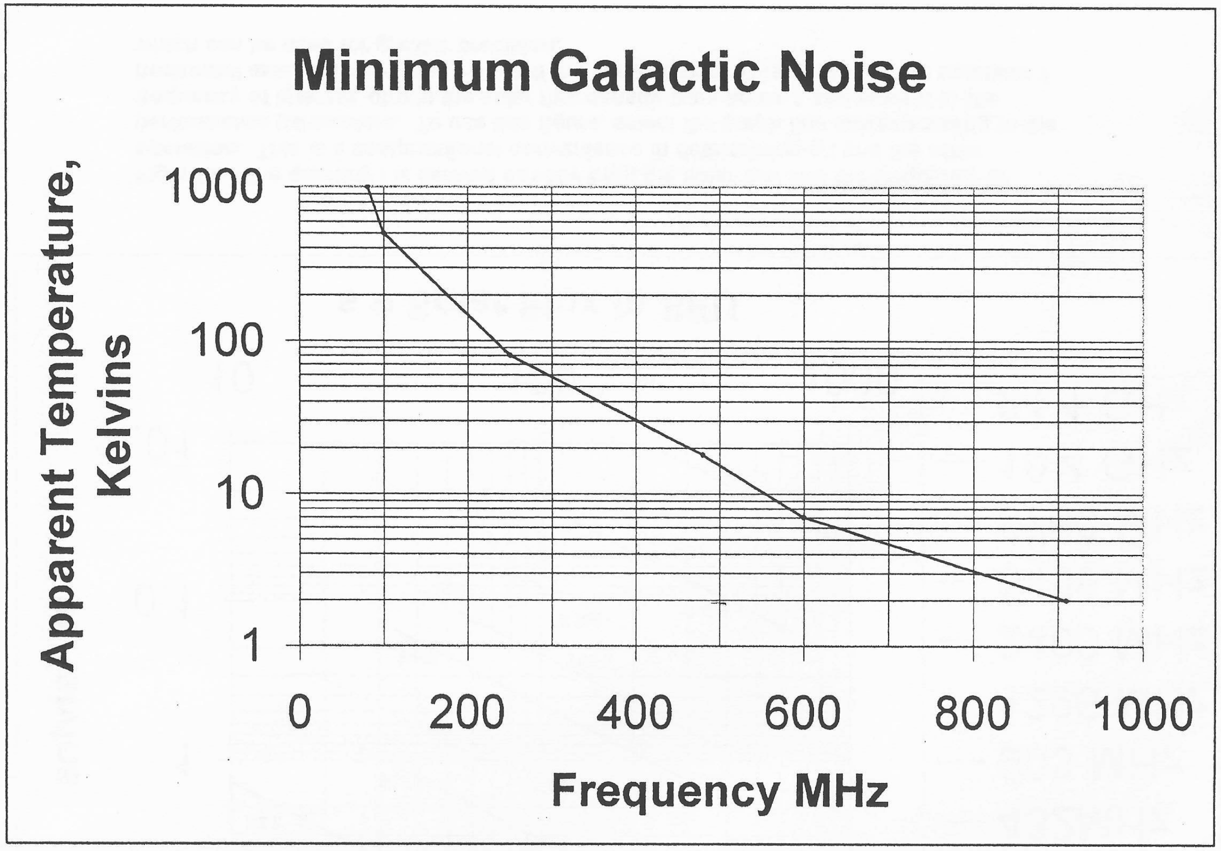
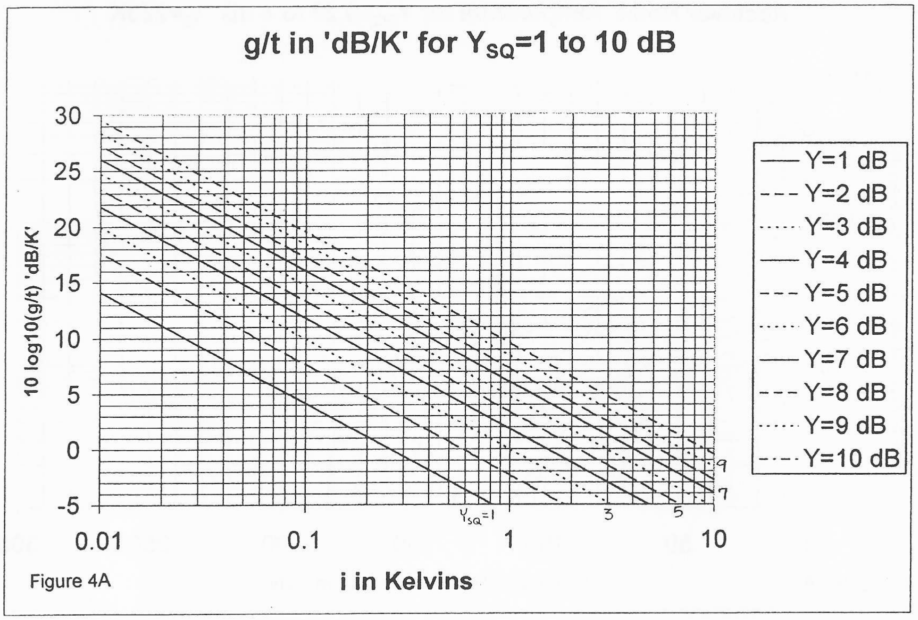


Figure 3 - For frequencies below 1000 MHz, our Milky Way Galaxy places a lower limit on the quiet-sky noise temperature. The value shown corresponds to most quiet portions of the sky. Above about 1000 MHz, this source of noise can be ignored, so long a one is not pointed toward the galactic plane.



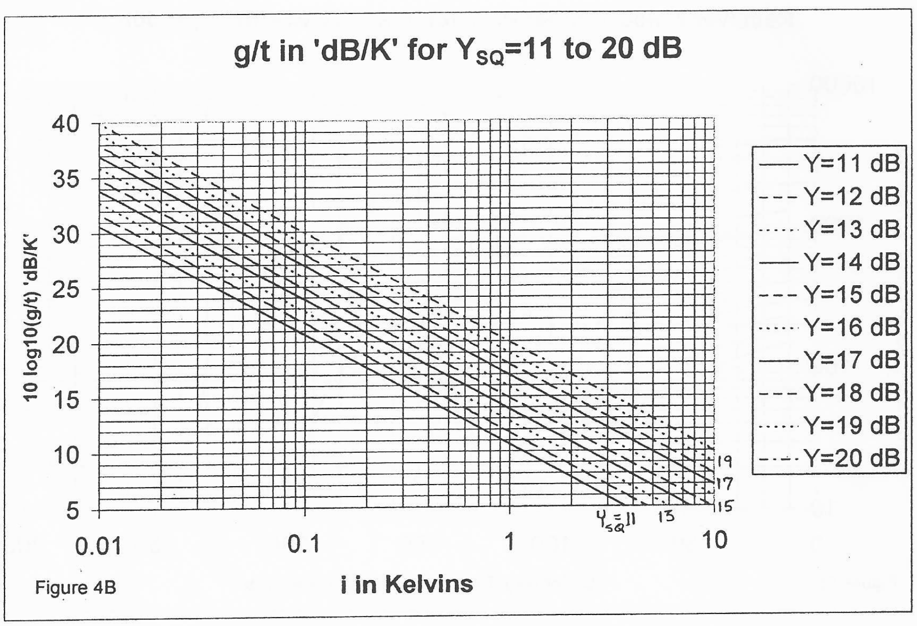


Figure 4, A and B - The Y_{SQ} value in these two plots is the difference in noise powers measured with the antenna pointed towards the Sun and the quiet sky, in dB. The *i* values come from Figure 2 or equation 1. This results in a *g/t* value, in dB. This is a measure of the overall receiving sensitivity, suitable for comparing different systems.

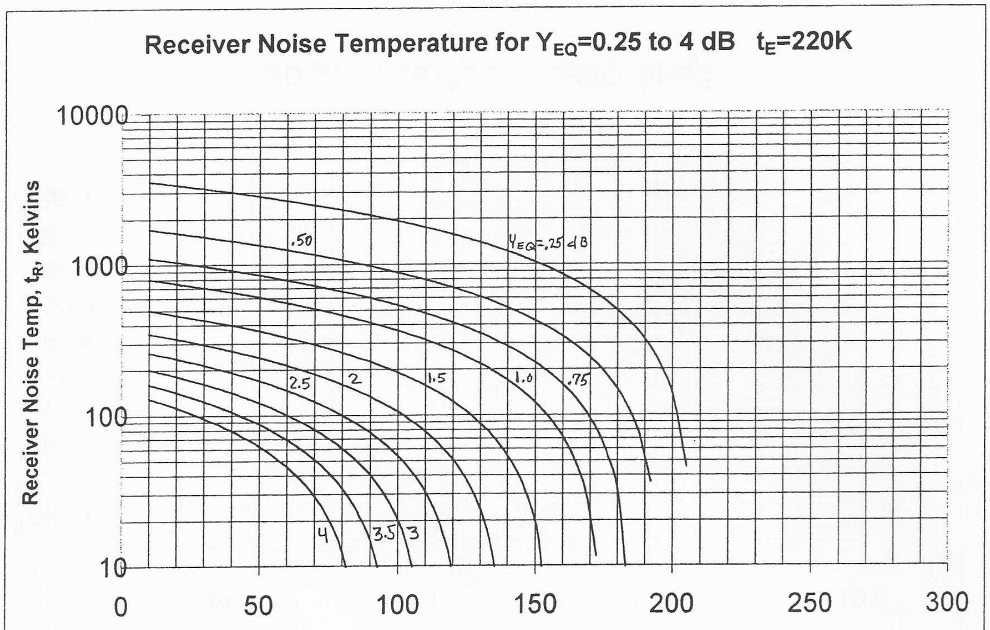


Figure 5A

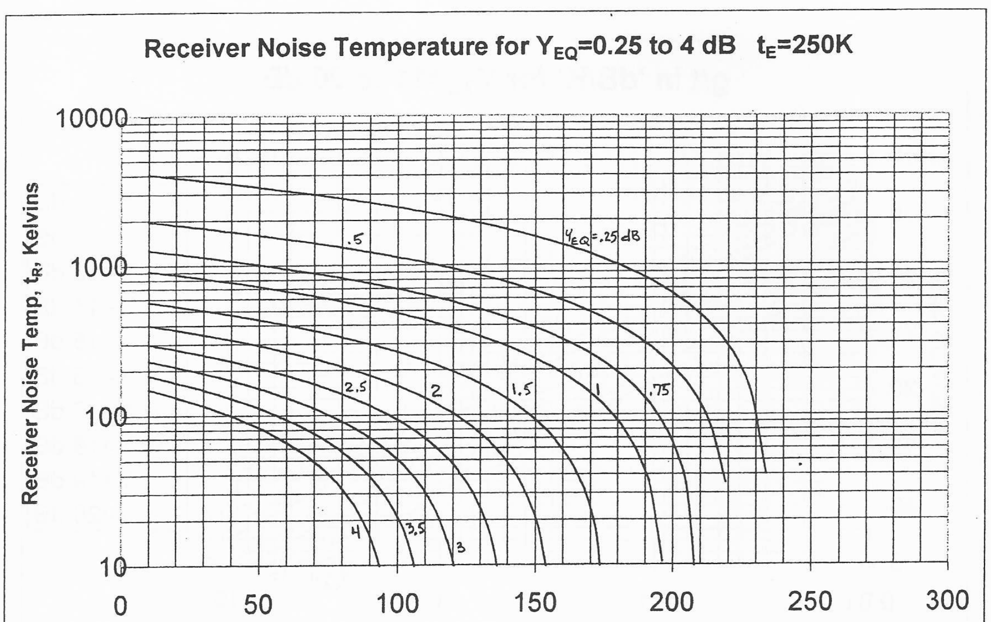


Figure 5B

Figure 5, A to C - The *Y_{EQ}* value in these two plots is the difference in noise powers measured with the antenna pointed towards the Earth and the quiet sky, in dB. This single measurement allows an estimate of the receiver noise temperature. As used here the receiver noise temperature includes noise due to antenna and feed-line losses as well as conventional front-end noise. The total system noise temperature is found by adding the receiver and quiet-sky noise temperatures. The graphs shown are Earth temperatures of 220, 250 and 290 K. Answers for any value can be calculated from equation 3 in the text, or by use of the Basic program in Appendix I.

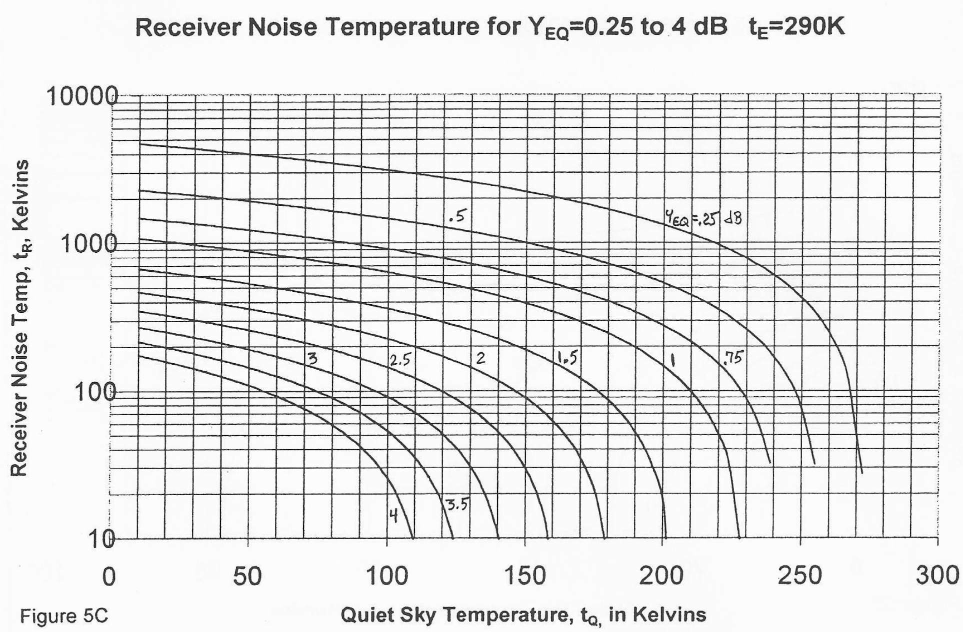


Figure 5C

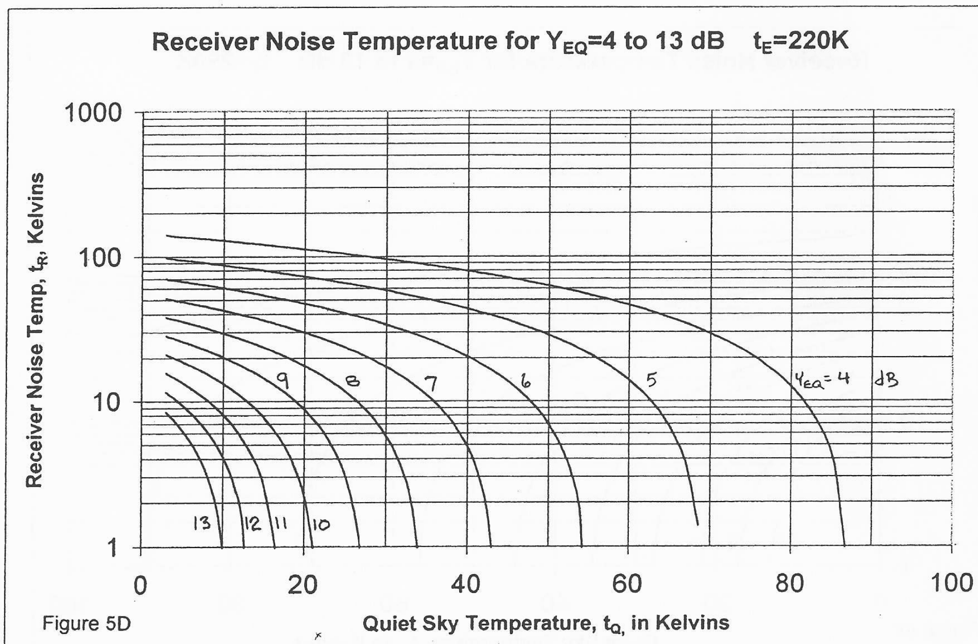


Figure 5D

Receiver Noise Temperature for Y_{EQ}=4 to 13 dB t_E=250K

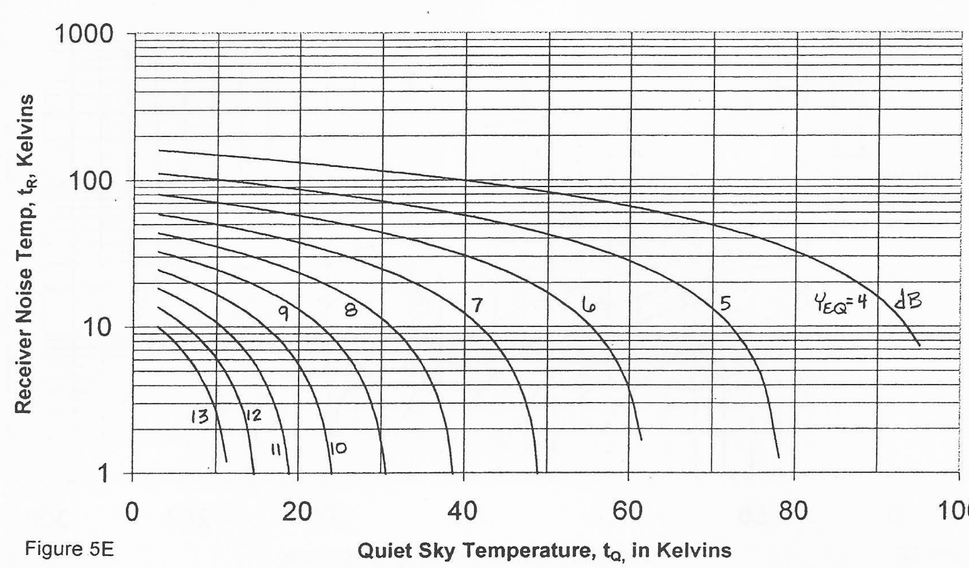


Figure 5E

Receiver Noise Temperature for Y_{EQ}=4 to 13 dB t_E=290K

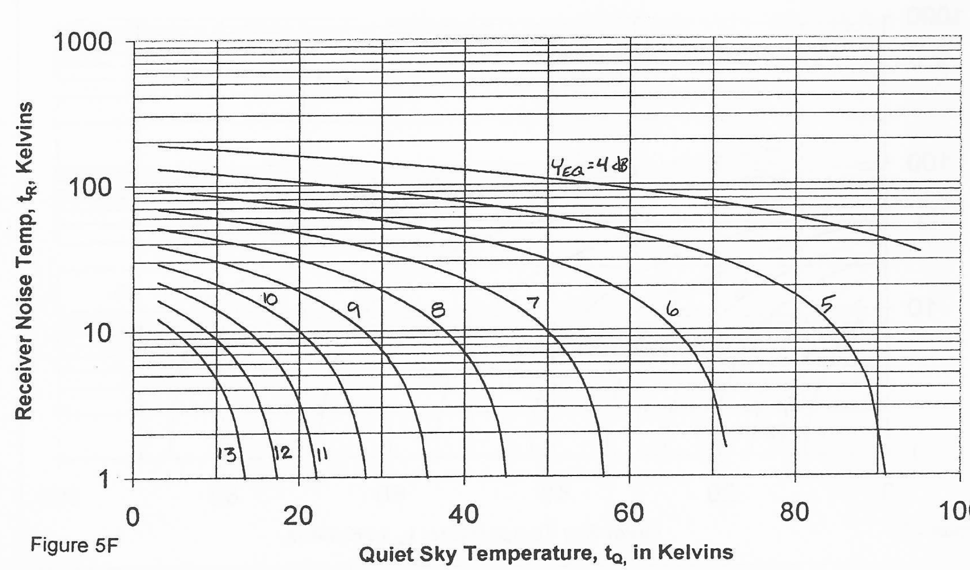


Figure 5F

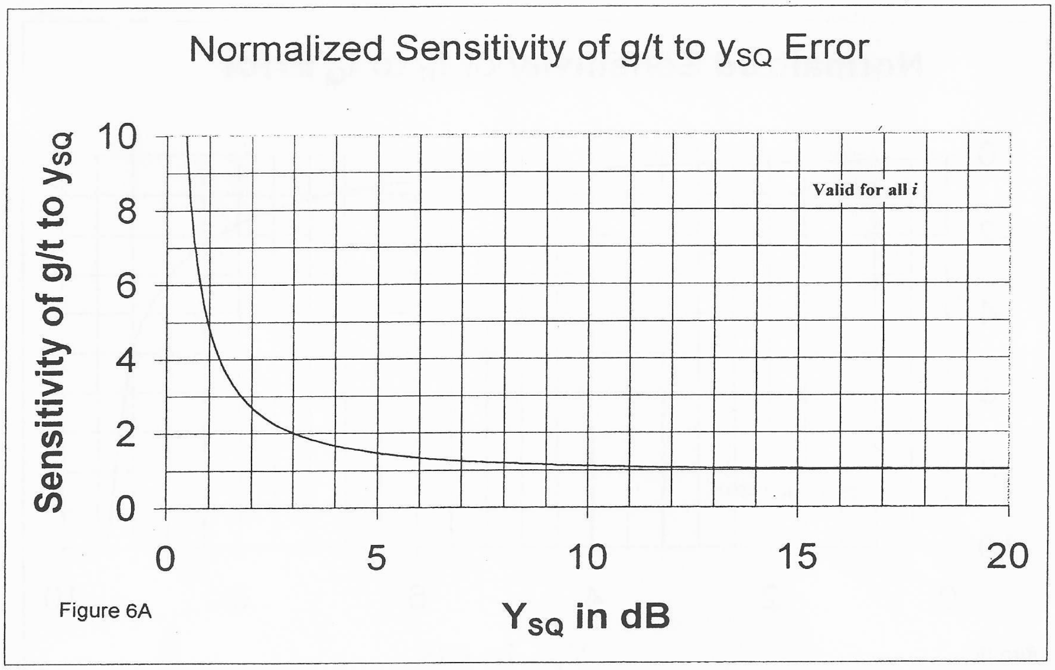


Figure 6A

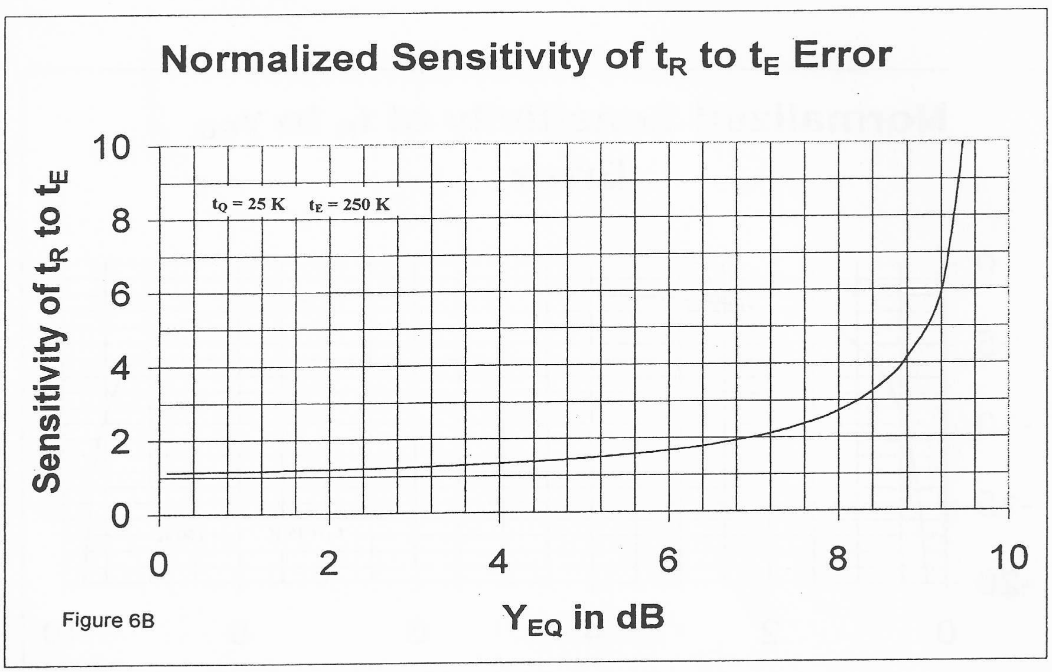


Figure 6B

Figure 6, A to D - Normalized sensitivities show the relationship between small fractional errors in the measured parameters and the resulting fractional error in estimated parameters, such as *g/t*, t_R and *g*. These normalized sensitivity curves cover some common ranges of values. The general cases can be calculated from the appendix equations A1-6 through A1-12, which also appear in the Basic program.

Normalized Sensitivity of t_R to t_Q Error

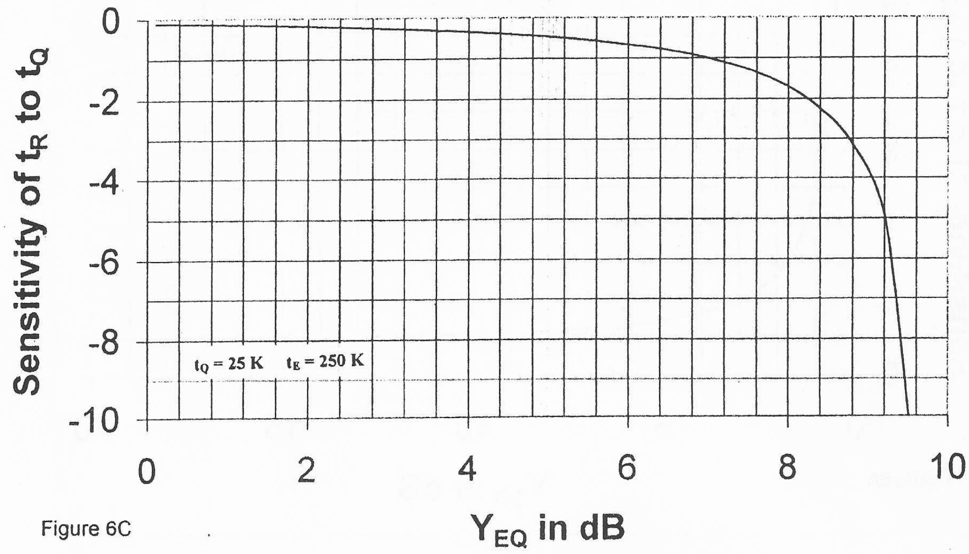


Figure 6C

Normalized Sensitivity of t_R to y_{EQ} Error

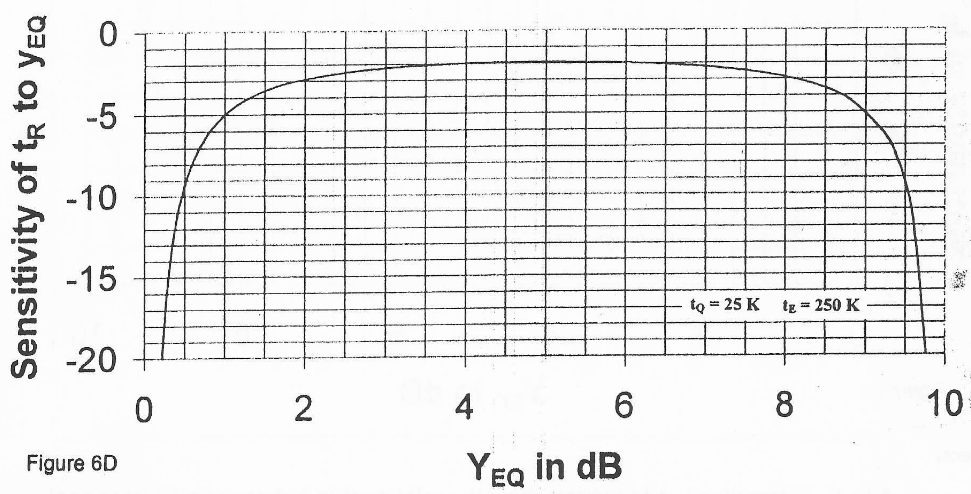


Figure 6D